

# **Networks and stability**

Part 1A. – Network topology

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- 1. network topology**
2. network dynamics
3. examples for networks
4. **synthesis** (complex equilibria,  
games, network evolution,  
trans-network effects)

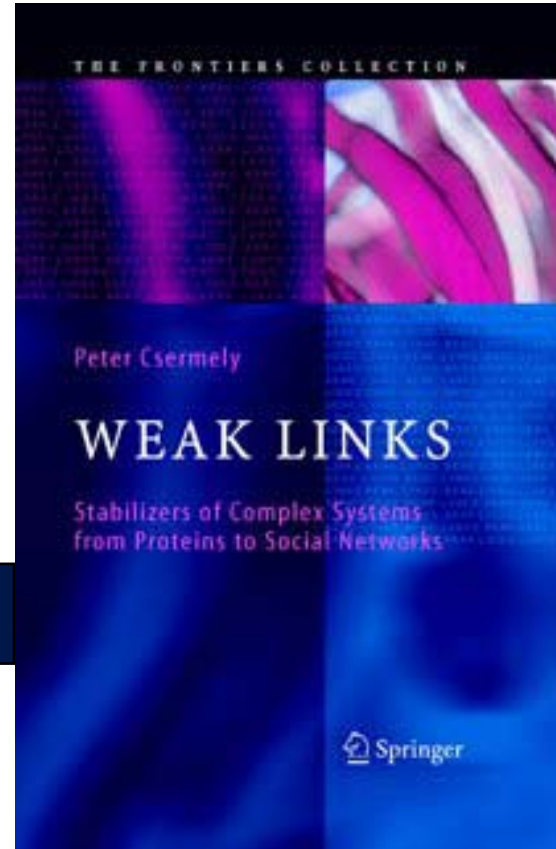
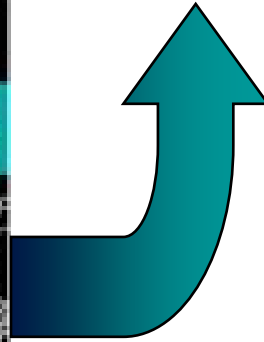
- 1. network topology (II.20-27.)**
- 2. network dynamics (III.6.-13.)**  
(III.20.-27. no lectures)
- 3. examples for networks (IV.3.-10.)**  
(IV.17. Easter)
- 4. synthesis (IV.24., V.1. holiday, V.8.)**  
**(V.13. consultation)**

[www.weaklink.sote.hu](http://www.weaklink.sote.hu)



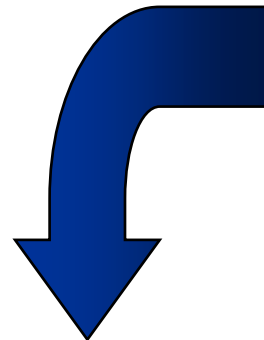
Hungarian

Vince Publisher, Budapest, 2005



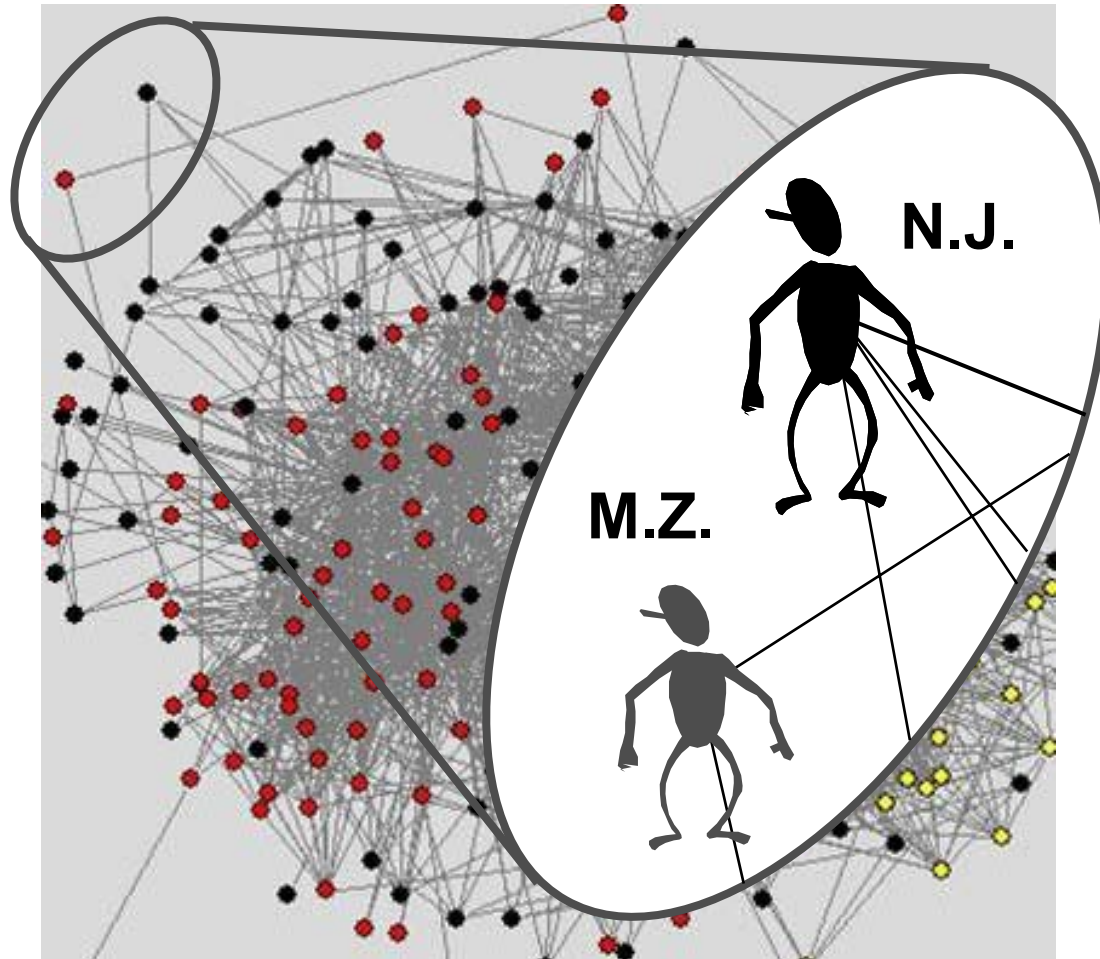
English

Springer, 2006

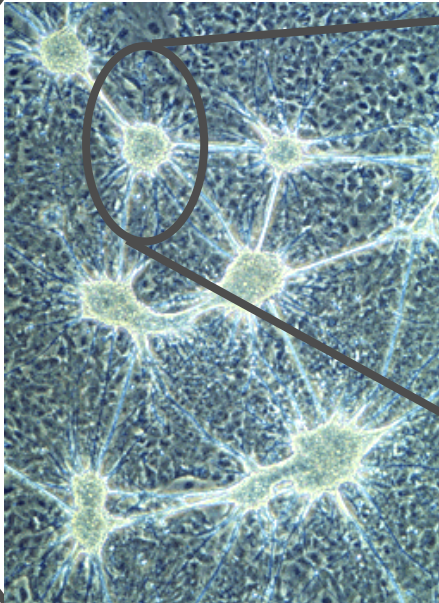


[www.weaklink.sote.hu](http://www.weaklink.sote.hu)

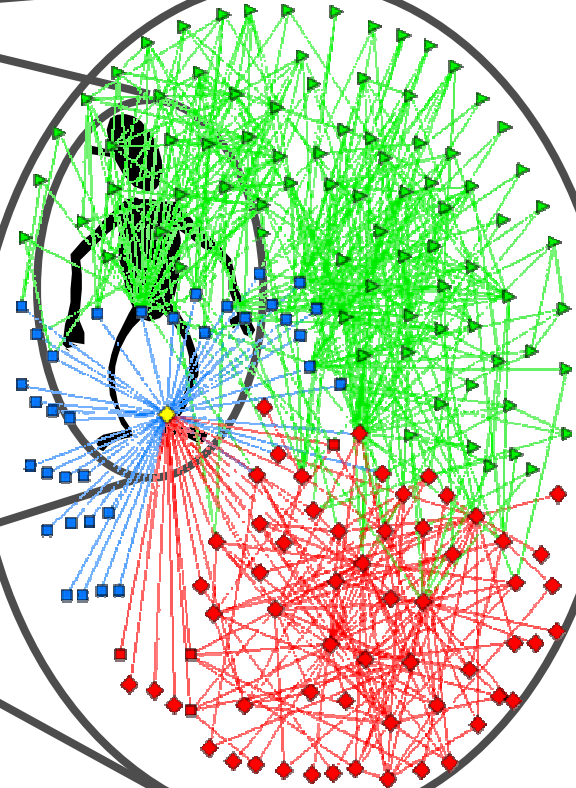
# Networks are embedded



# Networks are embedded

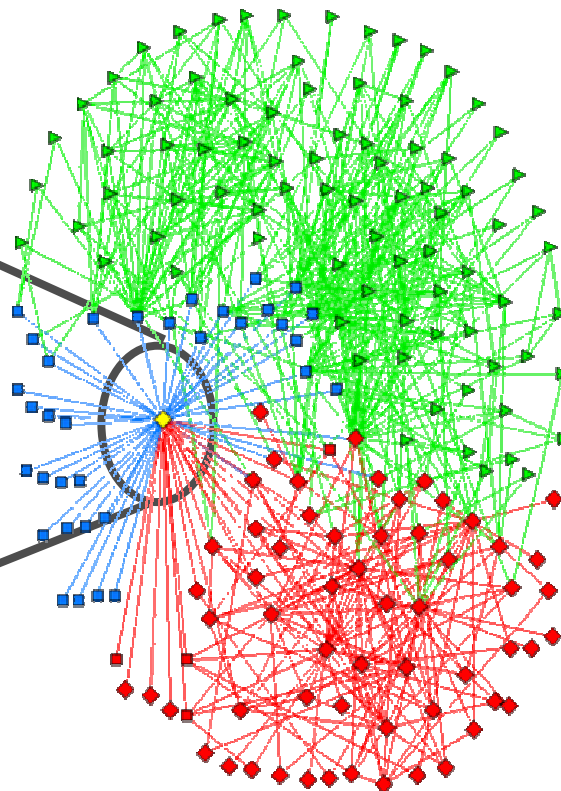
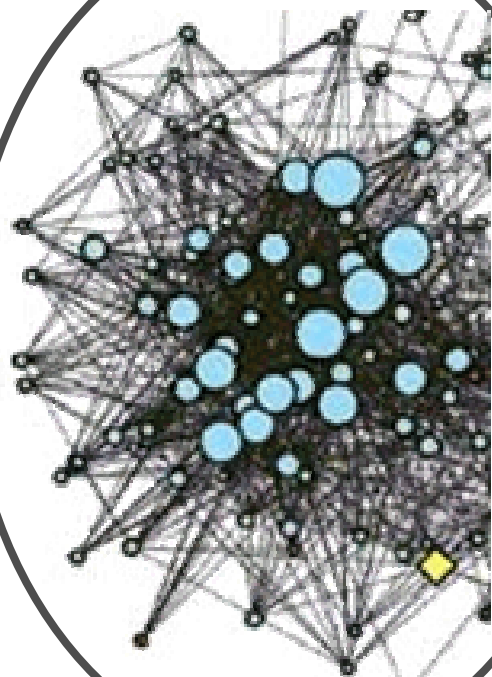


cell-net



protein-net

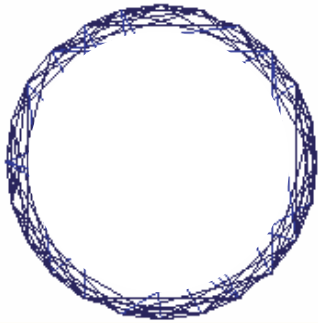
# Networks are embedded



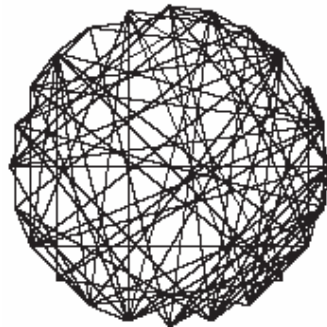
# Network topology

- **small worlds**
- scale-free degree distribution
- network communities
- network skeleton
- hierarchy and nestedness

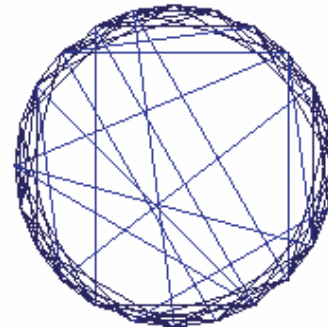
# Major classes of network topology



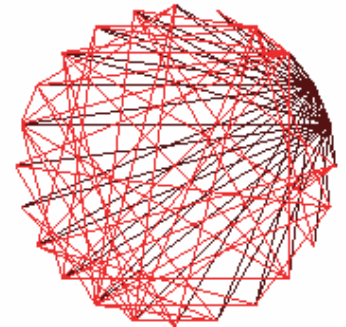
regular



random



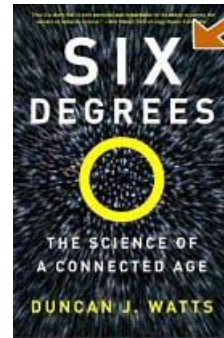
**small-world**



**scale-free**

# The Milgram-experiment

96 participants from Nebraska,  
1 target in Boston,  
18 letter chains via friends  
(first-name-basis)  
Psych. Today 1, 62; 1967  
Six degrees of separation

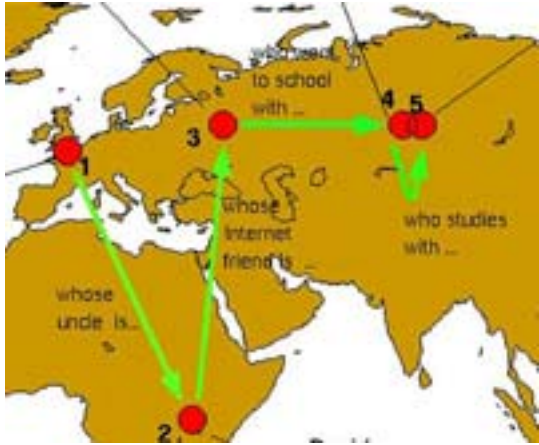


$100^6 = >100 \times$

the total population  
of the Earth

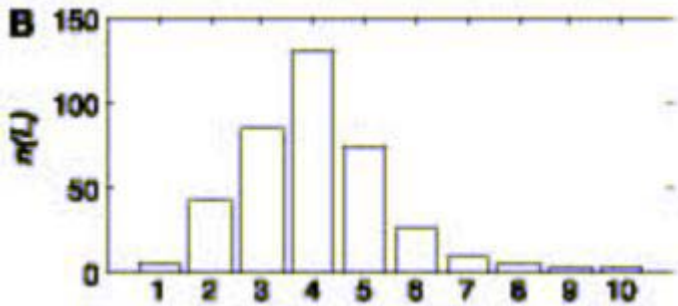


Frigyes Karinthy  
(1929) Five degrees



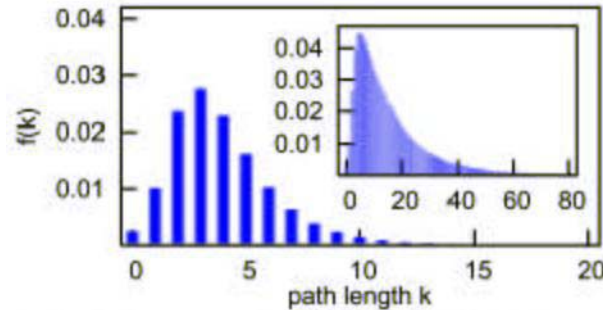
60,000 participants,  
 166 countries,  
 18 targets,  
 384 email chains  
 Science 301, 827; 2003

# Repeated Milgram- experiments

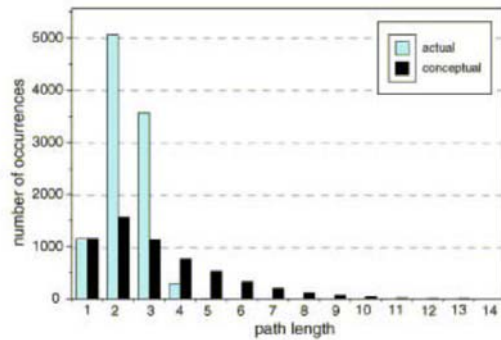


4 degrees  
 (5-7 if corrected  
 for drop-outs)

www.livejournal.com  
 500,000 US participants,  
 500,000 trials  
 PNAS 102, 11623; 2005



# Milgram-experiment: background



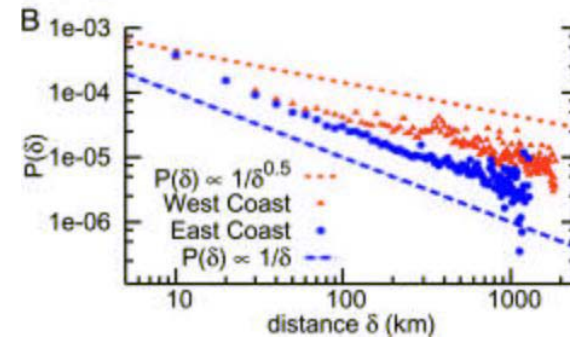
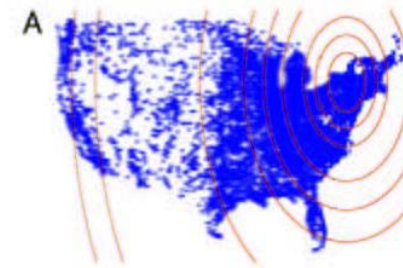
**52% wrong guesses**  
 Social Networks 28, 85; 2006

Table 1. Comparison of four *E. coli* network analyses

	Jeong et al. (7), directed	Wagner and Fell (5), undirected	Ma and Zeng (8), directed	This study, (un)directed
Top 10 hubs	H <sub>2</sub> O ADP P ATP L-glutamate NADP <sup>+</sup> PP NAD <sup>+</sup> NADPH NADH	L-glutamate pyruvate CoA $\alpha$ -keto glutarate L-glutamine L-aspartate acetyl CoA phosphoribosyl PP tetrahydrofolate succinate	glycerate 3P D-ribose 5P acetyl CoA pyruvate D-xylulose 5P D-fructose 6P 5P-D-ribose 1 PP L-glutamate D-glyceraldehyde 3P L-aspartate	carbon dioxide pyruvate acetyl CoA ATP D-glucose L-glutamate D-galactose CoA S-adenosyl L-methionine D-5-phosphoribosyl-1 P
AL	3.2	3.8	8.2	8.4 (8.0)

The top 10 hub metabolites and ALs reported in each study. Wagner and Fell (5) computed several versions of the network; the one shown here is the substrate-based network where ATP, ADP, NAD, NADP, NADH, NADPH, carbon dioxide, ammonia, sulfate, thioredoxin, (ortho)phosphate (P), and pyrophosphate (PP) are removed.

**avg. length may grow to 8**  
 PNAS 101, 1543; 2004



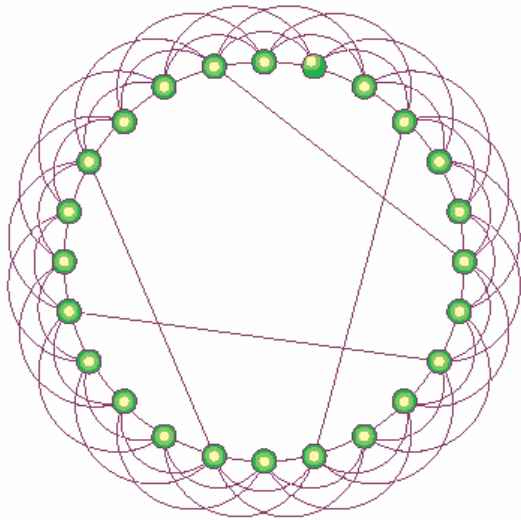
**geographic distance**  
**directs friendship circles**  
 PNAS 102, 11623; 2005

**+ social dimensions**

# Expansion of the small-world concept



Duncan Watts   Steve Strogatz



Nature 393, 440; 1998

high clustering coefficient  
AND small characteristic path length

general model

examples:

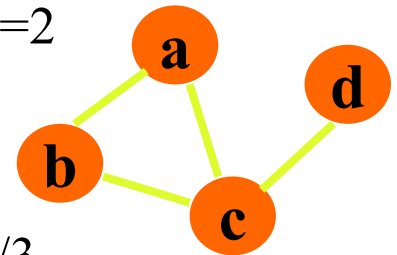
- *C. elegans* neurons
- US power grid
- film actor collaboration net

# Path length and clustering

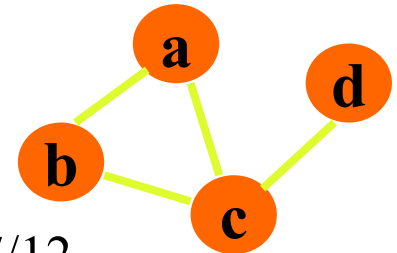
**Characteristic path length:**  
average of shortest pathlengths

**Random graphs: small**

- $D(ab)=1, D(ac)=1, D(ad)=2$
- $D(bc)=1, D(bd)=2$
- $D(cd)=1$
- $L=(1+1+2+1+2+1)/6 = 4/3$



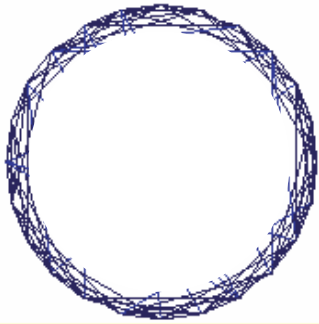
- $C(a)=C(b)=1$
- $C(c)=1/3$
- $C(d)=0$
- $C_{avg}=(1+1+1/3+0)/4 = 7/12$



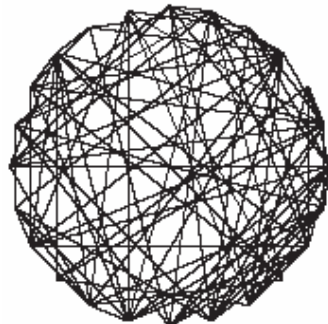
**Clustering coefficient:**  
connection of neighbors

**Regular lattices: large**

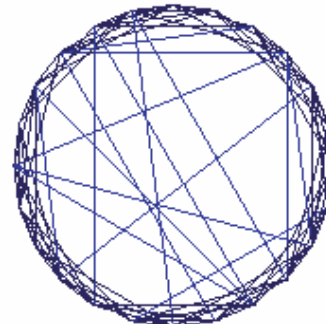
# Major classes of network topology



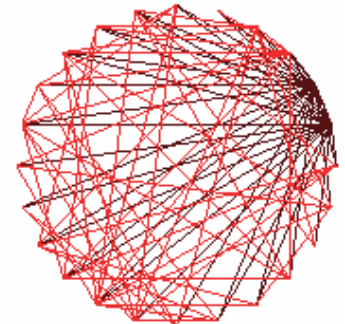
regular



random



**small-world**



**scale-free**

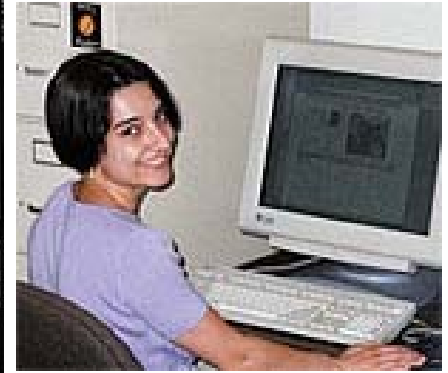
**the small world network gives  
low cost global connections**



# Network topology

- small worlds
- **scale-free degree distribution**
- network communities
- network skeleton
- hierarchy and nestedness

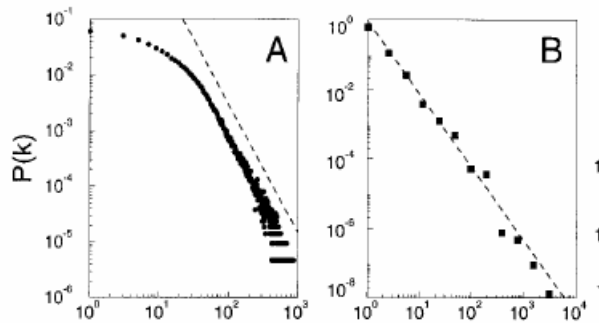
# Scale-free degree distribution



László Barabási

Réka Albert

degree-distribution



network model:

preferential attachment

(Matthew-effect, Pareto-law)

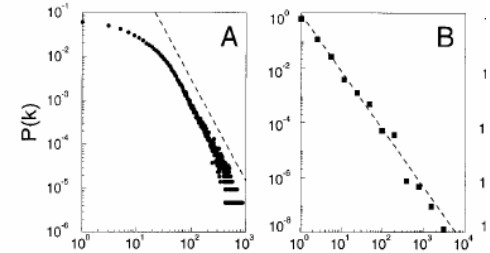
generality for actors, power grid, www

Science 286, 509; 1999

$$P = a k^{-\alpha} \quad (P \text{ probability, } a \text{ constant, } k \text{ degree, } \alpha \text{ exponent})$$

$$\lg P = \lg a - \alpha \lg k$$

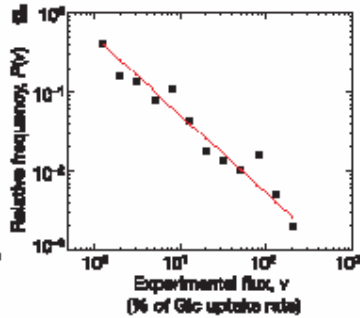
# How to make networks with scale-free degree distribution?



- preferential attachment
  - variable attractivity (fitness)
  - winner takes all (star-net)
  - aging networks (random net)
- duplication and divergence

# Generality of scale-free distributions

link-strength

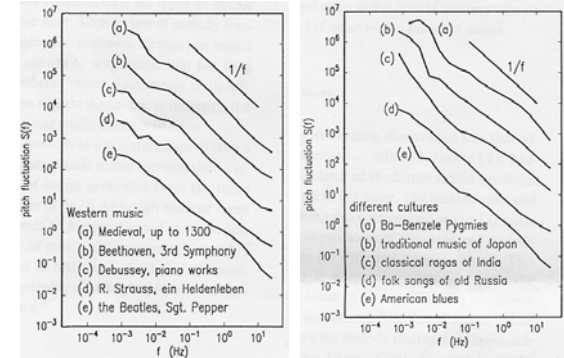


Nature 427, 839; 2004

Levy-flights

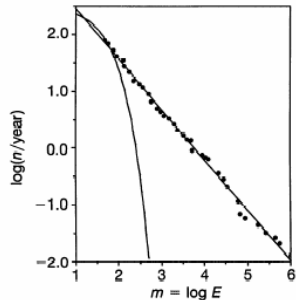


music



Nature 258, 317; 1975

probability, Noe-effect Can J. Zool. 80, 436; 2002



PNAS 92, 6689; 1995

Kohlrusch, 1854

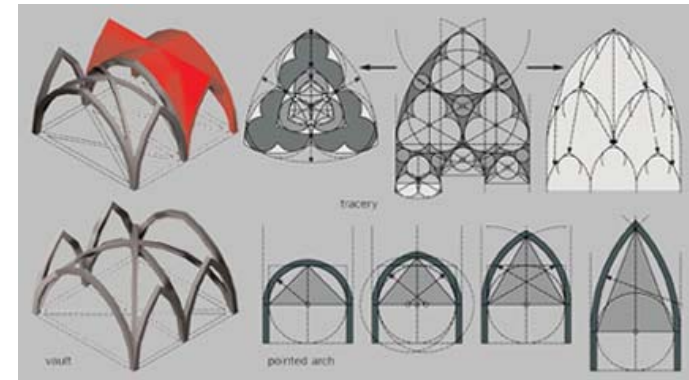
Leiden-jars

cumulative wins

Bernoulli 1738

town size Zipf-law  
 rain, lightning, tic  
 sexual contacts  
 earthquakes,  
 Gutenberg-Richter law  
 science papers  
 Lotka-law

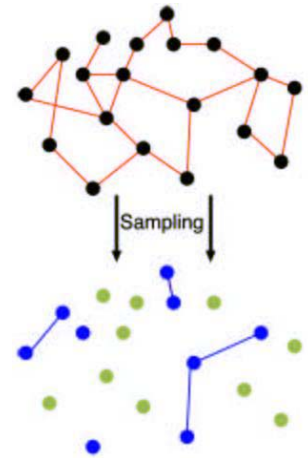
fractals, architecture



[www.iemar.tuwien.ac.at/modul23/Fractals](http://www.iemar.tuwien.ac.at/modul23/Fractals)

# Dangers of scale-free distributions

- must span many scales  
(network must be large enough)
- a line can be fitted to many curves...  
(log-normal, gamma, stretched exponential)
- cumulative plots are much better
- sampling bias
- unspecific data may cover real data
- distinct parts of networks are different



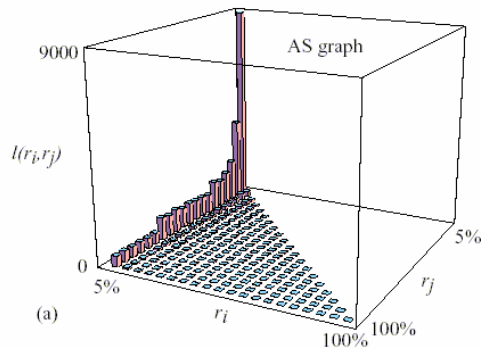
PNAS 102, 4221; 2005

# **Reasons behind the generality of scale-free distributions**

- preferential attachment
- cumulative success of consecutive tasks
- self-organization of matter in the Universe

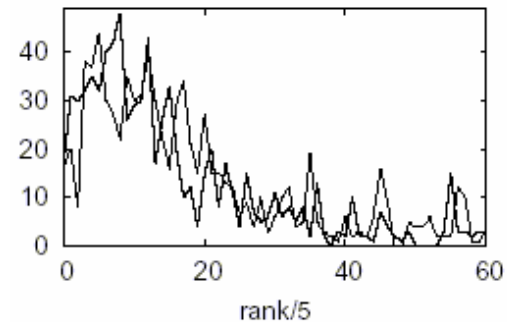
# Network clubs

**Rich-club:** hubs  
associate with hubs  
(assortativity: social nets)



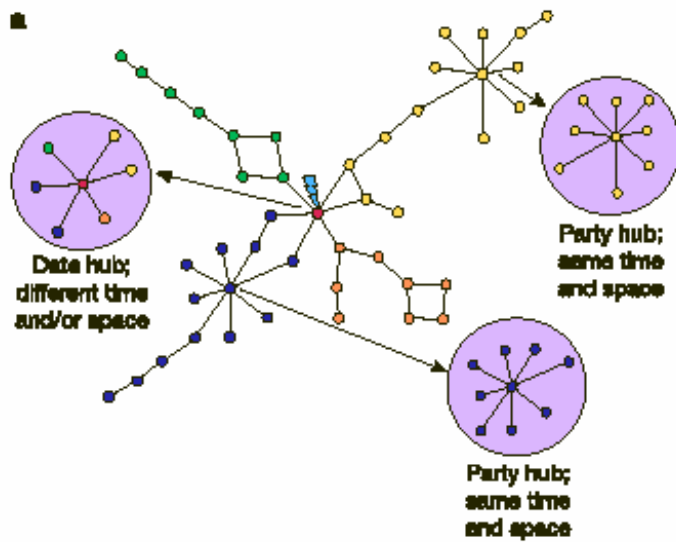
Zhou and Mondragon  
IEEE Comm. Lett. 8, 180

**Talented(VIP)-club:**  
isolated top rank  
associates with hubs



Masuda and Konno  
Social Networks, in press

# Date hubs and party hubs



Han et al. Nature 430, 88



yeast  
protein  
network

–date  
hubs

–party  
hubs