Networks and stability

Part 4. – Synthesis

Peter Csermely

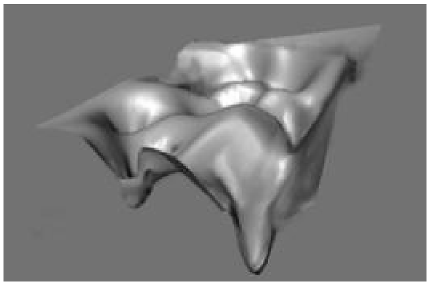
www.weaklink.sote.hu

1. network topology
2. network dynamics
3. examples for networks
4. **synthesis** **(complex equilibria, games, network evolution, trans-network effects)**

Synthesis

- **Le Chatelier principle of networks and complex equilibria**
- network catch-22 and 4 escape routes
- the Reward
- game theory of networks

Weak links smooth rough energy surfaces

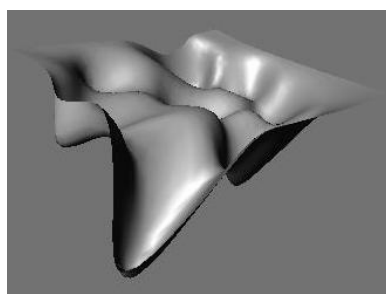


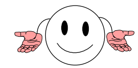
x-y plane: network parameters

(*not* emergent properties!)

z axis: energy, stability

weak links



Network comparison 

The network has **many** weak links	The network has **few** weak links
Well-connected	Sparsely connected
Bottom networks are optimally synchronized	Bottom networks are tightly coupled or behave independently
Relaxation is good	**Relaxation is disturbed, avalanches**
Noise is dissipated	Noise stays.
The network is **integrated**	The network is **segregated**
Errors are isolated	Error-prone
Saddles are transiently lowered	The transition is difficult.
Chances to find the minimal energy state are high.	**Large segments of the parameter space of the stability landscape are inaccessible** for the network.
The network is plastic.	The network is rigid.

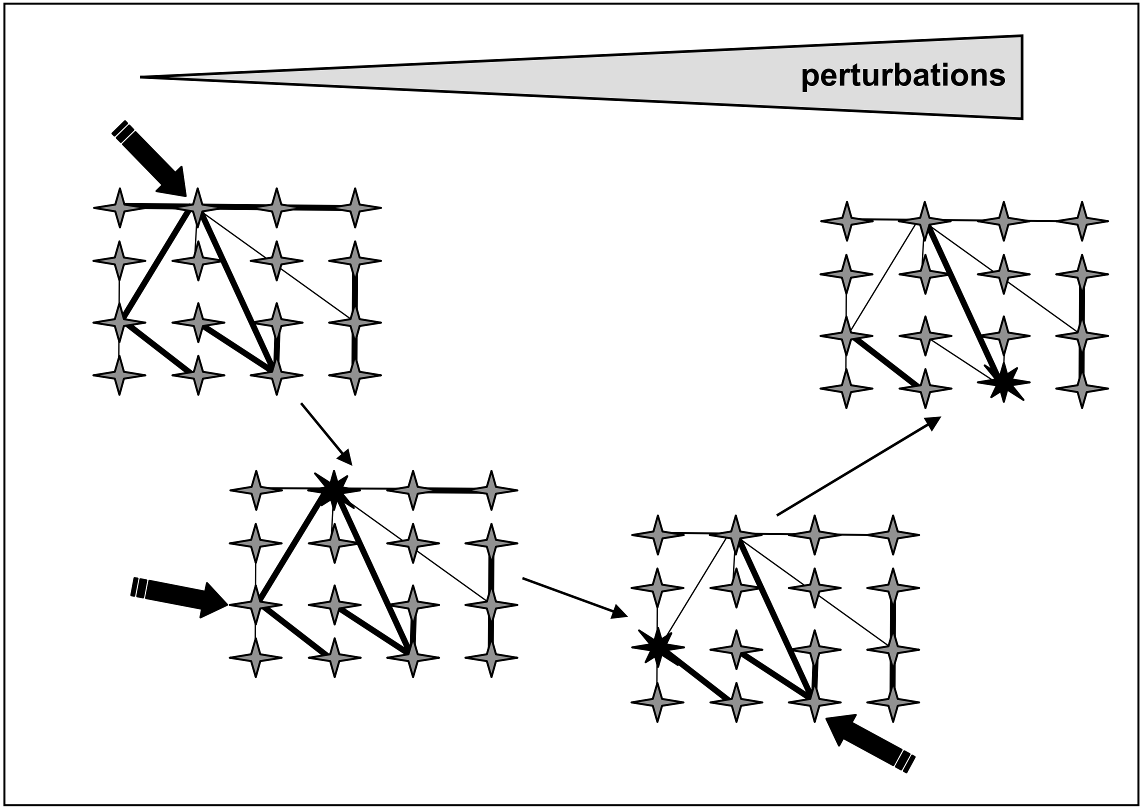
Weak links make punctuated equilibria less punctuated

Nature 366, 223

network	punctuated equlibr.	stability
protein	protein quake	energy
evolution	evolutionary jump	fitness
technology	innovation	originality
science	paradigm-change (Kuhn)	impact
firms	reorganization	value
economy	cycles (Schumpeter)	???

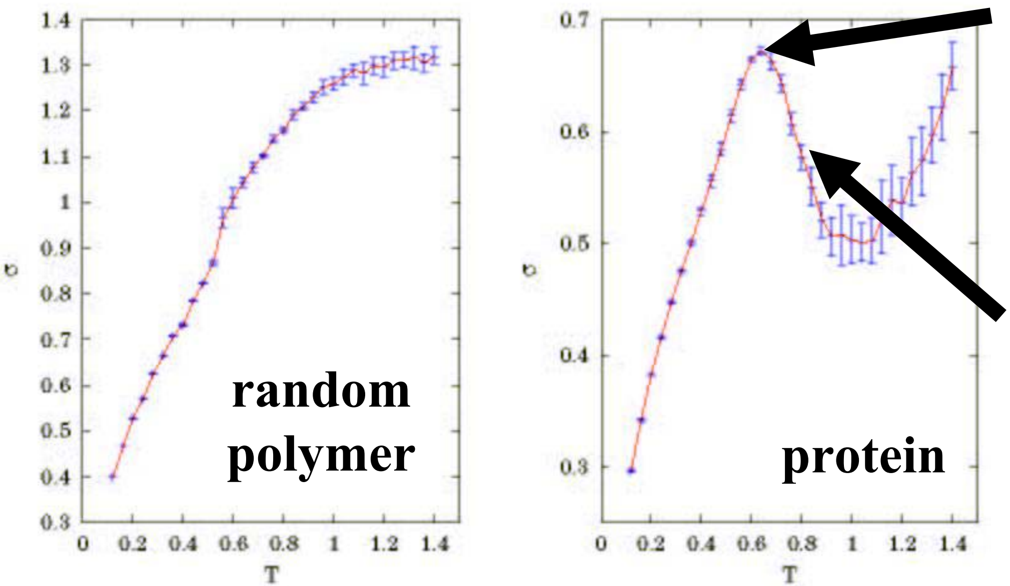
self-organized criticality (Bak)

Le Chatelier principle of networks: self-stabilization



Landscape fluctuations

Energy net fluctuation



maximal flexibility at native state

kinetic barrier against denaturation

Temperature

www.arxiv.org/cond-mat/0603409

Time scale!

What is unchanged and „rough”
at the minute/day timescale
may violently change and smooth
at the million-year timescale

Thermodynamics

Here we examine networks

• networks have non-identical elements

• networks have strong links

the classical thermodynamical description is useless

(n.b. the classical probability description

is useless as well)

Synthesis

- Le Chatelier principle of networks
  and complex equilibria
- **network catch-22 and 4 escape routes**
- the Reward
- game theory of networks

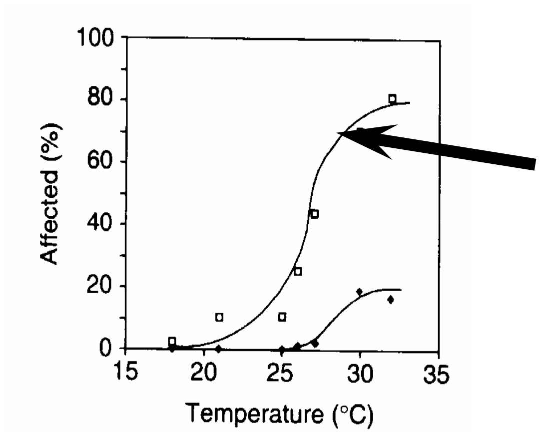
Complex organisms are trapped in their rough stability surface

Escape from this catch-22:

- local stability island extension

  (error-safe → diversity is allowed)
- jumps
- optimal jumps: Levy flights
- weak link modulation

Jumps in a rough stability surface: Chaperones buffer diversity



Rutherford and Lindquist, Nature 396, 336
Drosophila Hsp70, Oecologia 121, 323
Arabidopsis Hsp90, Nature 417, 618
E. coli Hsp60, Nature 417, 398
yeast Hsp90, Science 309, 2185

174 of 10,226 Hsp90 mutant *Drosophilas* suffer 23 types of malformations

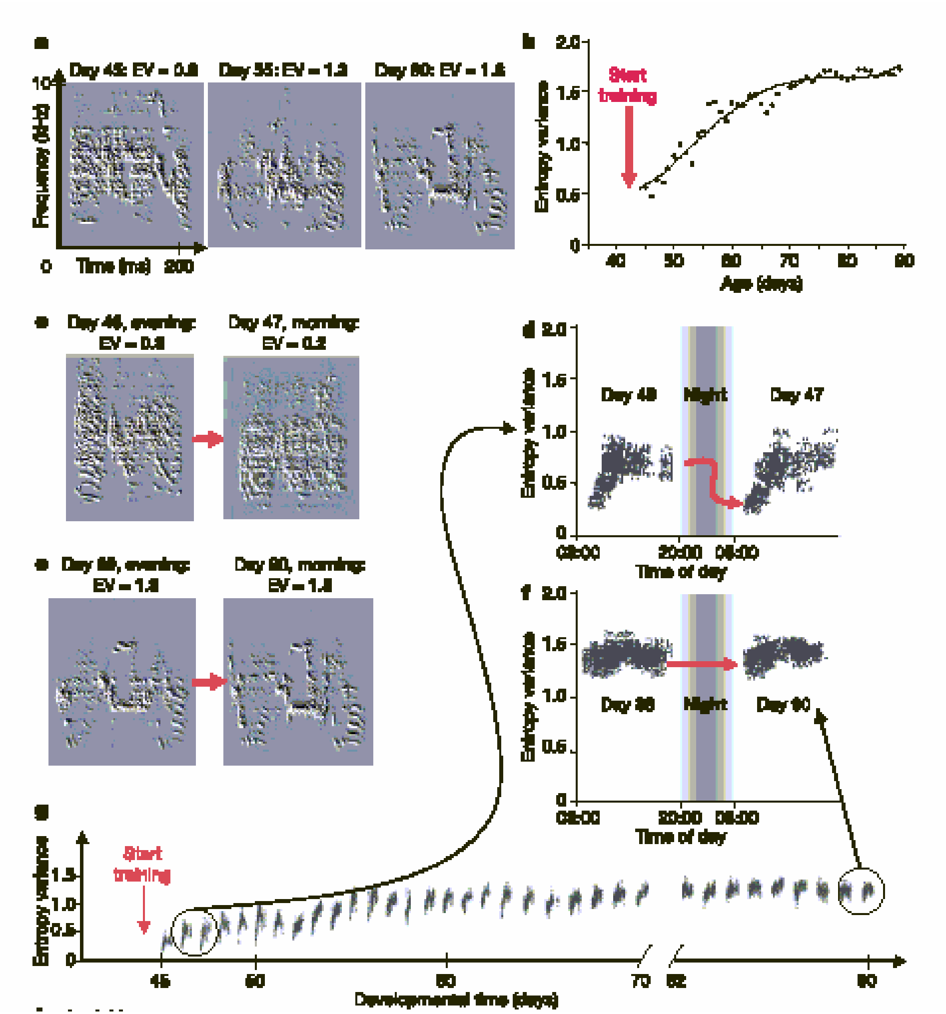
Complex organisms are trapped in their rough stability surface

Escape from this catch-22:

- local stability island extension

  (error-safe → diversity is allowed)
- jumps
- optimal jumps: Levy flights
- weak link modulation

Weak link modulation: Focused and fuzzy periods are needed for the real success



- brainstorming
- operationalization

bird-sleep erases part
of their songs

Nature 433, 710

Synthesis

- Le Chatelier principle of networks and complex equilibria
- network catch-22 and 4 escape routes
- **the Reward**
- game theory of networks

What is the reward?

stability

element	z-axis
protein	energy
evolution	fitness
firms	value

What happens?

selection

What happens NOW?

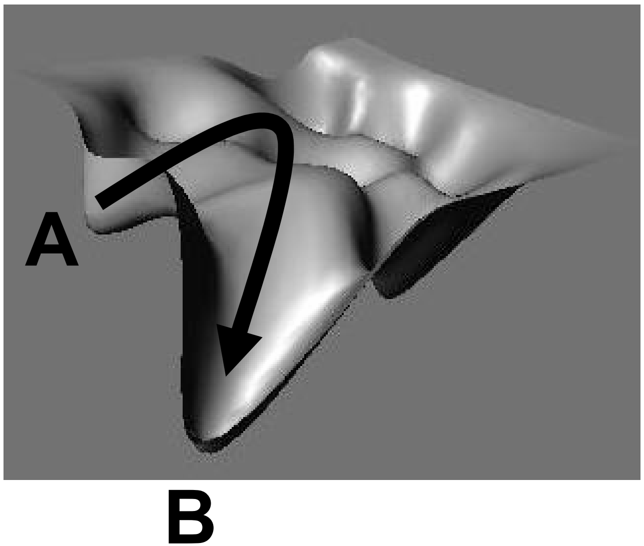
self-organization

Gaia?

Synthesis

- Le Chatelier principle of networks
  and complex equilibria
- network catch-22 and 4 escape routes
- the Reward
- **game theory of networks**

Stability landscapes CHANGE: game theory



change A → B provokes a change of the other elements → the landscape also changes!

cognitive limits → require games as simplified landscape-sets

28 lines of a Nobel-prize

This follows from the arguments used in a forthcoming paper.[13] It is proved by constructing an "abstract" mapping cylinder of λ and transcribing into algebraic terms the proof of the analogous theorem on CW-complexes.

* This note arose from consultations during the tenure of a John Simon Guggenheim Memorial Fellowship by MacLane.

[2] Whitehead, J. H. C., "Combinatorial Homotopy I and II," *Bull. A.M.S.*, 55, 214–245 and 453–496 (1949). We refer to these papers as CH I and CH II, respectively.

[3] By a complex we shall mean a connected CW complex, as defined in §5 of CH I. We do not restrict ourselves to finite complexes. A fixed 0-cell $e^0 \epsilon K^0$ will be the base point for all the homotopy groups in K.

[4] MacLane, S., "Cohomology Theory in Abstract Groups III," *Ann. Math.*, 50, 736–761 (1949), referred to as CT III.

[5] An (unpublished) result like Theorem 1 for the homotopy type was obtained prior to these results by J. A. Zilber.

[6] CT III uses in place of equation (2.4) the stronger hypothesis that λB contains the center of A, but all the relevant developments there apply under the weaker assumption (2.4).

[7] Eilenberg, S., and MacLane, S., "Cohomology Theory in Abstract Groups II," *Ann. Math.*, 48, 326–341 (1947).

[8] Eilenberg, S., and MacLane, S., "Determination of the Second Homology . . . by Means of Homotopy Invariants," these PROCEEDINGS, 32, 277–280 (1946).

[9] Blakers, A. L., "Some Relations Between Homology and Homotopy Groups," *Ann. Math.*, 49, 428–461 (1948), §12.

[10] The hypothesis of Theorem C, requiring that ν^{-1} (1) not be cyclic, can be readily realized by suitable choice of the free group X, but this hypothesis is not needed here (cf. [6]).

[11] Eilenberg, S., and MacLane, S., "Homology of Spaces with Operators II," *Trans. A.M.S.*, 65, 49–99 (1949); referred to as HSO II.

[12] $C(\tilde{K})$ here is the $C(K)$ of CH II. Note that $\tilde{K}$ exists and is a CW complex by (N) of p. 231 of CH I and that $p^{-1}K^n = \tilde{K}^n$, where p is the projection $p: \tilde{K} \rightarrow K$.

[13] Whitehead, J. H. C., "Simple Homotopy Types." If $W = 1$, Theorem 5 follows from (17:3) on p. 155 of S. Lefschetz, *Algebraic Topology*, (New York, 1942) and arguments in §6 of J. H. C. Whitehead, "On Simply Connected 4-Dimensional Polyhedra" (*Comm. Math. Helv.*, 22, 48–92 (1949)). However this proof cannot be generalized to the case $W \neq 1$.

EQUILIBRIUM POINTS IN N-PERSON GAMES

BY JOHN F. NASH, JR.*

PRINCETON UNIVERSITY

Communicated by S. Lefschetz, November 16, 1949

One may define a concept of an n-person game in which each player has a finite set of pure strategies and in which a definite set of payments to the n players corresponds to each n-tuple of pure strategies, one strategy being taken for each player. For mixed strategies, which are probability

distributions over the pure strategies, the pay-off functions are the expectations of the players, thus becoming polylinear forms in the probabilities with which the various players play their various pure strategies.

Any n-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the n strategy spaces of the players. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the $n - 1$ strategies of the other players in the countered n-tuple. A self-countering n-tuple is called an equilibrium point.

The correspondence of each n-tuple with its set of countering n-tuples gives a one-to-many mapping of the product space into itself. From the definition of countering we see that the set of countering points of a point is convex. By using the continuity of the pay-off functions we see that the graph of the mapping is closed. The closedness is equivalent to saying: if $P_1, P_2, \ldots$ and $Q_1, Q_2, \ldots, Q_n, \ldots$ are sequences of points in the product space where $Q_n \rightarrow Q$, $P_n \rightarrow P$ and Q_n counters P_n then Q counters P.

Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani's theorem[1] that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium point.

In the two-person zero-sum case the "main theorem"[2] and the existence of an equilibrium point are equivalent. In this case any two equilibrium points lead to the same expectations for the players, but this need not occu in general.

* The author is indebted to Dr. David Gale for suggesting the use of Kakutani' theorem to simplify the proof and to the A. E. C. for financial support.

[1] Kakutani, S., *Duke Math. J.*, 8, 457–459 (1941).

[2] Von Neumann, J., and Morgenstern, O., *The Theory of Games and Economic Behaviour*, Chap. 3, Princeton University Press, Princeton, 1947.

*REMARK ON WEYL'S NOTE "INEQUALITIES BETWEEN TH. TWO KINDS OF EIGENVALUES OF A LINEAR TRANSFORMATION"**

BY GEORGE POLYA

DEPARTMENT OF MATHEMATICS, STANFORD UNIVERSITY

Communicated by H. Weyl, November 25, 1949

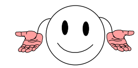
In the note quoted above H. Weyl proved a Theorem involving a fun tion $\varphi(\lambda)$ and concerning the eigenvalues α_i of a linear transformation and those, κ_i, of A^*A. If the κ_i and $\lambda_i = |\alpha_i|^2$ are arranged in descendi: order,

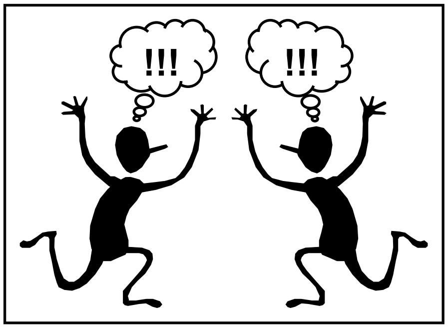
Nash equilibrium (1950)

Nobel-prize of economy 1994

(with John Harsányi and Reinhardt Selten)

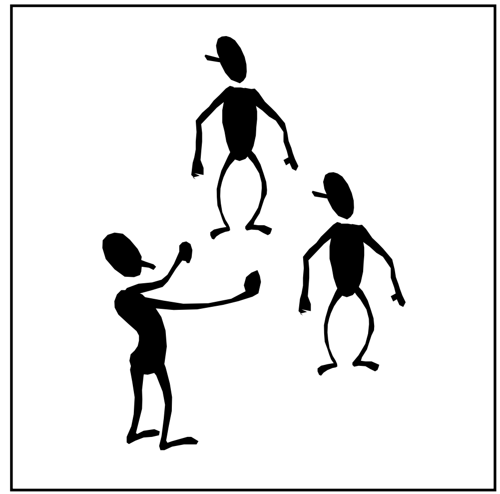
Weak links make the game-net continuous





rigid
game-rules

weak
links



convergent
game-rules

Hierarchy

Hierarchy of nets.
How many networks do we have?

1. network of players
2. a single player as a network
3. network of stability points (equilibrium conditions)
4. network of transitions (games)